Section 3.1 – The Three Forms of Linear Equations

This booklet belongs to:_____ Block: _____

Standard Form and General Form of a Linear Equation

- If $A, B, and\ C$ are real numbers, the equation $\boldsymbol{Ax + By = C}$ is called the **STANDARD FORM** of the equation of a line.
- It is **best to write** the equation with $A, B, and\ C$ **as integers**, and $\boldsymbol{A \geq 0}$ (Not Negative).
- **Standard Form** was introduced in Grade 9 and won't be used much longer
- The **GENERAL FORM**: $\boldsymbol{Ax + By + C = 0}$ is a more appropriate form since moving forward we will want our **Linear Equations equal to $\boldsymbol{0}$**
- So, the **transformation** from **Standard to General** Form is a simple one:
    - Get **ALL THE TERMS** on the **same side** of the **equal sign**

Standard Form	**General Form**
$\boldsymbol{Ax + By = C}$ $A, B, C\ are\ Integers,\quad A > 0$	$\boldsymbol{Ax + By + C = 0}$ $A, B, C\ are\ Integers,\quad A > 0$

Example 1: Write the following in **General Form**

$-3x + y = 4$ and $\frac{2}{3}x + 2y = 3$

Solution 1:

$\boldsymbol{-3x + y = 4}$ can be expressed as: $3x - y = -4$ ← multiply all terms by (-1)

$3x - y = -4$ can be expressed as: $\boldsymbol{3x - y + 4 = 0}$ ← add 4 to both sides

$\boldsymbol{\frac{2}{3}x + 2y = 3}$ can be expressed as: $2x + 6y = 9$ ← multiply each term by (3)

$2x + 6y = 9$ can be expressed as: $\boldsymbol{2x + 6y - 9 = 0}$ ← subtract 9 from both sides

Slope-Intercept Form of a Linear Equation

- The equation $\boldsymbol{y = mx + b}$ is the **SLOPE-INTERCEPT FORM** of the equation of a line.
- The $\boldsymbol{y - intercept}$ of the line is $(\boldsymbol{0}, \boldsymbol{b})$, and the $\boldsymbol{slope}$ of the line is $\boldsymbol{m}$.
- The algebra of **STANDARD FORM** to **SLOPE-INTERCEPT FORM** is as follows:

$$Ax + By = C \quad \rightarrow \quad By = -Ax + C \quad \rightarrow \quad y = -\frac{A}{B}x + \frac{C}{B}$$

- The $\boldsymbol{slope}$ of $Ax + By = C$ is $-\frac{A}{B}$
- The $\boldsymbol{y - intercept}$ of $Ax + By = C$ is $\frac{C}{B} \rightarrow (0, \frac{C}{B})$

SLOPE-INTERCEPT FORM

$$\boldsymbol{y = mx + b}$$

Example 2: Convert $-2x - 3y = 12$ to **Slope-Intercept Form** and then Graph the Linear Equation

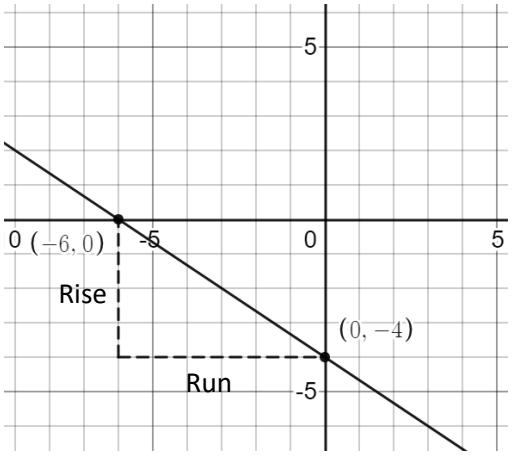
Solution 2:

$-2x - 3y = 12$ so… $-3y = 2x + 12$ so… $y = \frac{2x + 12}{-3}$

so… $\boldsymbol{y = -\frac{2}{3}x - 4}$

- The $\boldsymbol{slope}$ of the line is: $\boldsymbol{\frac{2}{3}}$ and the $\boldsymbol{y - intercept}$ is $(\boldsymbol{0}, \boldsymbol{-4})$

- Once you do the algebra it's easy to map
- Plot the y-intercept
- Trace your slope
- Map the next point
- Continue that process and connect the points with a line



Graphing a Line Using the $\boldsymbol{Slope}$ and the $\boldsymbol{y - Intercept}$

Step 1: Write the equation in **SLOPE INTERCEPT FORM** by solving for y
Step 2: Identify the $\boldsymbol{y - intercept}$ $(\boldsymbol{0}, \boldsymbol{b})$, and graph this point
Step 3: Graph a **second point using the $\boldsymbol{slope}$**, starting at the $y - intercept$
Step 4: Draw a line connecting the points to obtain the graph

Example 3:

Graph: $3x + 2y = 12$ by using the $slope\ and\ y - intercept$

Solution 3:

Step 1: $\boldsymbol{3x + 2y = 12}$ so… $\boldsymbol{2y = -3x + 12}$ so… $\boldsymbol{y = -\frac{3}{2}x + 6}$

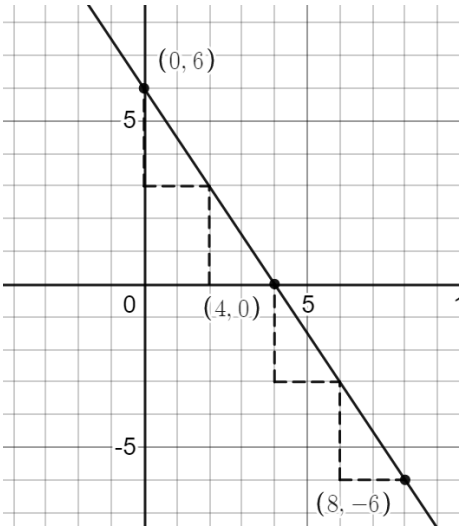
Step 2: The $y - intercept$ is $(0, 6)$: mark this point

Step 3: The slope is $m = \frac{rise}{run} = -\frac{3}{2}$

From $(0, 6)$, go $down\ 3$ units and to the $right\ 2$ units, to obtain the point $(2, 3)$.

Repeat that step as many times as you want.

Step 4: Draw a line through the points $(0, 6), (2, 3)$ and $(4, 0)$.



A Note About Slope

When you have a slope, it's a fraction.

So, if it's **negative**: $-\frac{3}{2} = \frac{-3}{2} = \frac{3}{-2}$

- You can go **DOWN and RIGHT** or **UP and LEFT**

So, if it's **positive**: $\frac{3}{2} = \frac{3}{2} = \frac{-3}{-2}$

- You can go **UP and RIGHT** or **DOWN and LEFT**

Graphing a Line Using the Slope and a Point

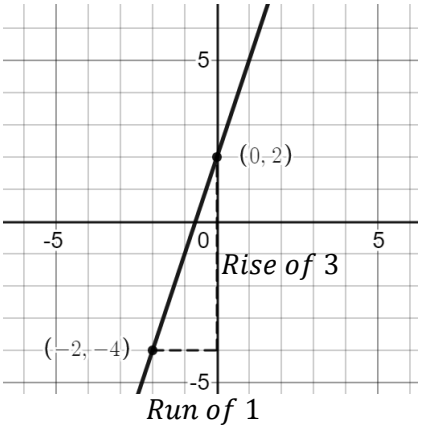
Step 1: Locate and graph the **given point**.
Step 2: Graph another point **tracing the slope**, counting from the first point
Step 3: Repeat that step again
Step 4: Draw a line connecting the three points to obtain the graph

Example 4:

Graph the line through $(-2,-4)$ with $slope\ \frac{3}{1}$.

Solution 4:

The $slope$ is 3, so from the point $(-2,-4)$ go $\boldsymbol{up\ 3\ units}$, and to the $\boldsymbol{right\ 1\ unit}$ to get the next point $(-1,-1)$. **Repeat tracing the slope** from the new point $(-1,-1)$.



Writing an Equation of a Line Using a Slope and a Point

- By **substituting** given **values** for a $\boldsymbol{slope\ and\ a\ point}\ (\boldsymbol{x},\boldsymbol{y})$ **of a line** into $y = mx + b$, the line's equation can be found!

Example 5:

Write the equation of the line with $slope\ 2$ that runs through $(-4,1)\ in\ \boldsymbol{slope\ intercept - form.}$

Solution 5:

The point $(-4,1)$ gives us an $x - value$ of -4 and a $y - value$ of 1.

So, $y = mx + b \rightarrow$ $1 = 2(-4) + b$

$1 = -8 + b$

$b = 9$

Therefore, the equation of the line is… $\boldsymbol{y = 2x + 9}$

Point-Slope Form of a Linear Equation

- The equation $y - y_1 = m(x - x_1)$ is the **POINT-SLOPE EQUATION** of a line.
- The **given point** is (x_1, y_1) and the **slope** of the line is m
- This formula comes from rearranging the definition of the slope, $m = \frac{y - y_1}{x - x_1}$

POINT-SLOPE EQUATION

$$\boldsymbol{y - y_1 = m(x - x_1)}$$

Example 6:

Write the equation of a line with $slope$ 2 that passes through $(-4, 1)$ in **Slope-intercept form**.

Solution 6: $y - y_1 = m(x - x_1) \rightarrow$

$$y - 1 = 2(x - (-4))$$

$$y - 1 = 2(x + 4)$$

$$y - 1 = 2x + 8$$

$$\boldsymbol{y = 2x + 9}$$

Example 7:

Write the equation of a line with $slope$ $\frac{4}{5}$ that passes through $(3, -2)$ in **Standard form.**

Solution 7: $y - y_1 = m(x - x_1) \rightarrow$

$$y - (-2) = \frac{4}{5}(x - 3)$$

$$y + 2 = \frac{4}{5}(x - 3)$$

$$5(y + 2) = 4(x - 3)$$

$$5y + 10 = 4x - 12$$

$$\boldsymbol{4x - 5y = 22}$$

Section 3.1 – Practice Problems

Complete each statement

1. The formula for the $\boldsymbol{Point - Slope\ form}$ of a line is _____
2. In the equation $y = mx + b$, $(\mathbf{0}, \boldsymbol{b})$ is called the _____
3. The equation $\boldsymbol{y = mx + b}$ is called the _____ form
4. The $\boldsymbol{Standard\ form}$ of the equation of a line is _____

Find the $\boldsymbol{slope\ and\ the\ y - intercept}$

5. $3x - 2y = 6$

$Slope$:

$Y - int$:

6. $4x + 3y = 12$

$Slope$:

$Y - int$:

7. $2x - 5y = -7$

$Slope$:

$Y - int$:

8. $5x + 2y = 0$

$Slope$:

$Y - int$:

9. $x - 4y = -4$

$Slope$:

$Y - int$:

10. $6x - y = -3$

$Slope$:

$Y - int$:

Rewrite the ***Standard Form Equation*** in ***Slope − Intercept Form***

11. $2x + y = 6$

12. $3x - y = 4$

13. $4x + 3y = 12$

14. $2x - 3y = 6$

15. $5x + 4y = 3$

16. $6x - 3y = 4$

Rewrite the ***Slope − Intercept Equation*** in ***Standard Form***

17. $y = -2x + 1$

18. $y = 3x - 1$

19. $y = 3x$

20. $y = -\frac{2}{3}x + 1$

21. $y = \frac{3}{4}x + 5$

22. $y = -\frac{2}{5}x + \frac{1}{2}$

Rewrite the $\boldsymbol{Point - Slope\ Equation}$ in $\boldsymbol{Slope - Intercept\ Form}$

23. $y - 2 = 3(x + 1)$

24. $y + 4 = -2(x - 1)$

25. $y - 1 = \frac{1}{3}(x + 2)$

26. $y + 4 = -\frac{2}{5}(x - 3)$

27. $y - \frac{2}{3} = \frac{1}{4}(x - 8)$

28. $y - \frac{1}{4} = \frac{1}{2}\left(x + \frac{2}{3}\right)$

Rewrite the $\boldsymbol{Point - Slope\ Equation}$ in $\boldsymbol{Standard\ Form}$

29. $y - 2 = 3(x + 1)$

30. $y + 4 = -2(x - 1)$

31. $y - 1 = \frac{1}{3}(x + 2)$

32. $y + 4 = -\frac{2}{5}(x - 3)$

33. $y - \frac{2}{3} = \frac{1}{4}(x - 8)$

34. $y - \frac{1}{4} = \frac{1}{2}\left(x + \frac{2}{3}\right)$

Write the equation of each line in $\boldsymbol{slope - intercept\ form}$

35. $(0, 2); m = 2$

36. $(0, -3); m = \frac{1}{2}$

37. $(0, 3); m = 0$

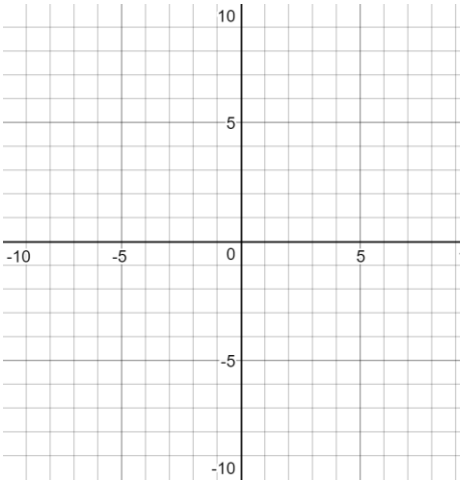
38. $(0, -2); m = -\frac{2}{3}$

39. $\left(0, -\frac{1}{2}\right); m = -\frac{3}{4}$

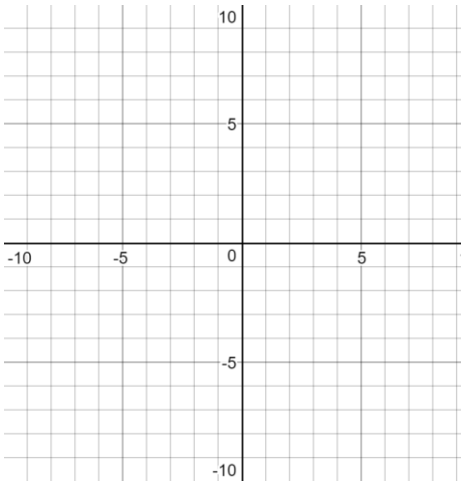
40. $(0, 2.3); m = 0.4$

Graph the linear Equation

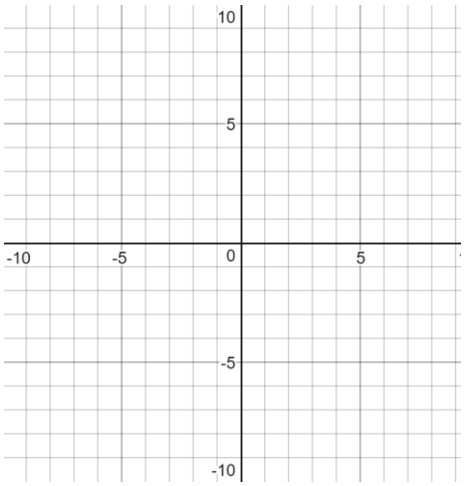
41. $4x - 3y = 12$



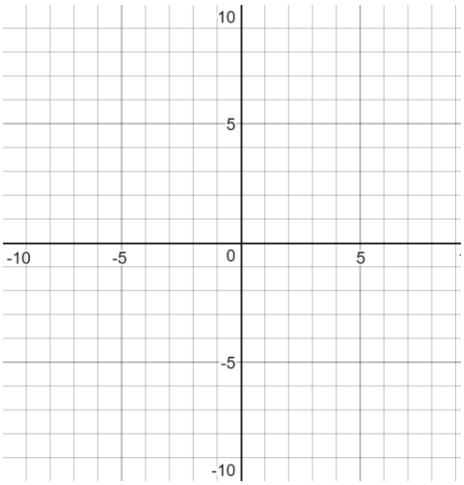
42. $y = -\frac{2}{3}x + 4$



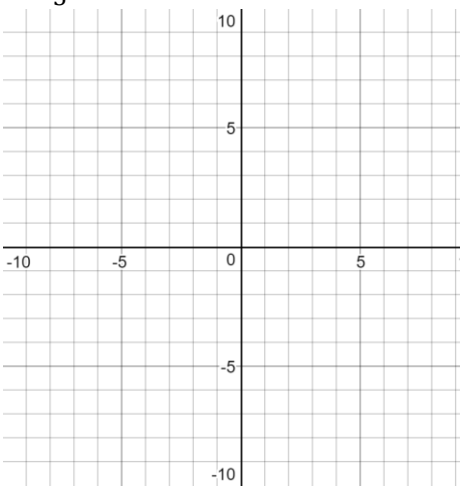
43. $y - 3 = \frac{1}{2}(x + 4)$



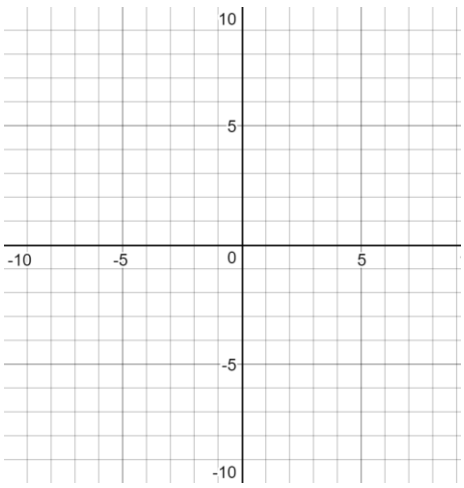
44. $2x + 3y = 10$



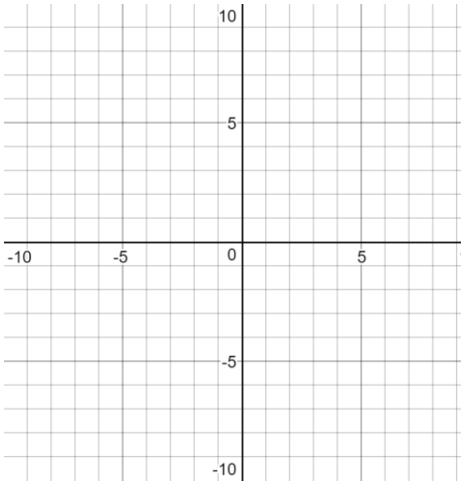
45. $y + 2 = -\frac{2}{3}(x + 5)$



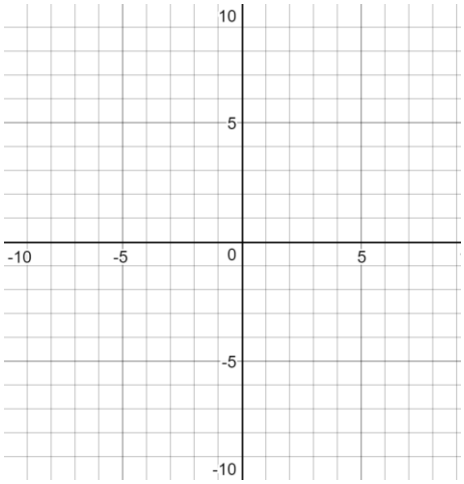
46. $5x - 2y = 0$



47. $y - \frac{5}{2} = -\frac{1}{2}\left(x + \frac{3}{2}\right)$



48. $y = \frac{5}{3}x - \frac{7}{2}$



Section 3.1 – Answer Key

1. $y - y_1 = m(x - x_1)$
2. $y - intercept$
3. $Slope - Intercept$
4. $Ax + By = C$
5. $Slope: \frac{3}{2}; y - int: -3$
6. $Slope: -\frac{4}{3}; y - int: 4$
7. $Slope: \frac{2}{5}; y - int: \frac{7}{5}$
8. $Slope: -\frac{5}{2}; y - int: 0$
9. $Slope: \frac{1}{4}; y - int: 1$
10. $Slope: 6; y - int: 3$
11. $y = -2x + 6$
12. $y = 3x - 4$
13. $y = -\frac{4}{3}x + 4$
14. $y = \frac{2}{3}x - 2$
15. $y = -\frac{5}{4} + \frac{3}{4}$
16. $y = 2x - \frac{4}{3}$
17. $2x + y = 1$
18. $3x - y = 1$
19. $3x - y = 0$
20. $2x + 3y = 3$
21. $3x - 4y = -20$
22. $4x + 10y = 5$
23. $y = 3x + 5$
24. $y = -2x - 2$
25. $y = \frac{1}{3}x + \frac{5}{3}$
26. $y = -\frac{2}{5}x - \frac{14}{5}$
27. $y = \frac{1}{4}x - \frac{4}{3}$
28. $y = \frac{1}{2}x + \frac{7}{12}$
29. $3x - y = -5$
30. $2x + y = -2$
31. $x - 3y = -5$
32. $2x + 5y = -14$
33. $3x - 12y = 16$
34. $6x - 12y = -7$
35. $y = 2x + 2$
36. $y = \frac{1}{2}x - 3$
37. $y = 3$
38. $y = -\frac{2}{3}x - 2$
39. $y = -\frac{3}{4}x - \frac{1}{2}$
40. $y = 0.4x + 2.3$
41. See Website
42. See Website
43. See Website
44. See Website
45. See Website
46. See Website
47. See Website
48. See Website

Extra Work Space